A Novel Convex Power Adaptation Strategy for Multicast Communications using Random Linear Network Coding Schemes

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***Abstract*—3GPP's Long Term Evolution (LTE) represents the one of the most valuable alternatives to offer a wireless broadband access in fully mobile network context. In particular LTE is able to manage several communication flows characterized by different QoS constrains. This paper deals with a network topology where the mobile users are clustered in Multicast Groups and the base station broadcasts a different traffic flow to each cluster. In order to improve the network throughput on a per-user basis, all communications rely on a Random Linear Network Coding (RLNC) scheme. A key aspect in the QoS management is represented by the power adaptation strategy in use. This paper proposes a novel convex formulation to the power adaptation problem for the downlink phase taking into account the specific RLNC scheme adopted by each communication flow. By the proposed convex formalization, an optimal solution of the problem can be early found in real time. Moreover, the proposed power adaptation strategy shows good performance for what concern throughput and fairness among the users when compared with other alternatives.**

I. Introduction

Several broadband wireless communication standards have been proposed to deliver Internet-based services to mobile users. In particular the 3GPP's Long Term Evolution (LTE) represents the latest and most promising OFDM-based communication standard by allowing to handle several uplink/downlink communication flows characterized by different QoS constraints in a Full-IP communication network.

In order to guarantee data integrity compliant with specific QoS profiles, it was proposed the use of Automatic Repeat-reQuest (ARQ) or Hybrid-ARQ (HARQ) schemes [1].

Moreover, recently the use of Network Coding (NC) schemes has been considered as an efficient alternative in order to reduce the ARQ/HARQ related overhead [2] and to improve the network performance in terms of throughput, delivery delay and packet corruption in the case of multicast communications [3]–[6]. The NC principle was presented in the seminar paper of Ahlswede *et al.* [7] with the aim of improving the capacity of wired networks. Subsequently, NC schemes were successfully considered also in wireless networks [8] in order to improve bandwidth exploitation in the presence of not reliable communication channels.

Among several NC approaches [9], this work deals with the Random Linear NC (RLNC) one [10] due to the reduced complexity of the coding operations [11], [12]. In a RLNC scheme each coded symbol (belonging to a given finite field) is obtained by a linear combination of a set of plain symbols using a vector of coefficients randomly chosen. For these reasons the probability that a coded symbol is linearly independent to the previous ones is not zero. A sink node needs to collect enough linearly independent symbols in order to decode the original message [13].

This paper foresees an application scenario where a base station (the eNodeB in LTE systems) has to send different information flows to specific groups of Users Equipments (UEs) called *Multicast Groups* (MGs) randomly located within the coverage area. Each node belonging to a group receives the same downlink communication flow from the eNodeB. The specific problem discussed here is that of defining an optimization approach able to allocate to each individual information flow the most suitable power level in order to improve throughput, the delivery delay and fulfil the transmission power constraint at the eNodeB side.

The problem of the optimal power adaptation has been investigated in several works [14], [15] but to the best of our knowledge, it has never been explicitly addressed taking into account that the downlink communications rely on a RLNC scheme. This papers deals with this problem by considering a novel convex optimization model implementing a power adaptation strategy able to lead to a fair downlink radio resource access among the UEs of each MG.

The organizations of this paper is the following: the description of the system model is presented in Sec. II, the NC-based power adaptation model is shown in Sec. III and Sec. IV presents the numerical results obtained by computer simulations. Finally conclusions are drawn in Sec. V.

II. System model

In this paper an OFDMA system for mobile wireless communications based on LTE standard is considered. It is important to note that the method proposed is suitable for both uplink and downlink communications. However, for the sake of simplicity, we focus here on the downlink communications only.

LTE can be operated both in time or frequency division duplexing but in this work we will focus on the TDD scheme due to its flexibility [16]. In a TD-LTE system (TDD version of LTE) the radio resource is organized in *radio frames* (10 ms long) as follows: each radio frame consists of two *half frames* of 5 ms, an half frame is in turn divided into five *sub-frames* (1 ms long). Each sub-frame can be assigned to the uplink or the downlink phase according to the specific uplink/downlink configuration adopted in the network. Seven different configurations can be used but we will refer to the sixth one [1] because this paper deals with the downlink radio resource allocation problem and this configuration is characterized by the biggest number of sub-frames allocated for the downlink phase.

The downlink sub-frames are split into fixed size elements called Physical Resource Blocks (PRBs). A PRB consists of 6 or 7 OFDM symbols (5.21 µs long) depending on the particular cyclic prefix adopted and of 12 contiguous subcarriers. The downlink radio resource can be modeled as a time/frequency matrix of $O \times S$ PRBs (i.e., the frequency and time domains are respectively split into O and S PRBs). Each traffic flow coming from the higher layers is split at the Medium Access Control (MAC) layer in atomic units called Protocol Data Units (PDUs). Each PRB holds data belonging to only one PDU.

The MAC layer plays a central role in the resource allocation and, as consequence, in the QoS management of each communication flow between the eNodeB and the UEs. The MAC layer should take into account different aspects such as the channel quality, the priority of each flow and the network traffic load. It is important to point out that the proposed method is independent on the policy used to allocate PBRs to different traffic flows.

In the rest of this section the considered network topology and communication strategy will be introduced.

A. Multicast communication model

We focus on a wireless network with one eNodeB and K MGs. Each MG is composed by W_k UEs, with $k = 1, \ldots, K$, that are randomly placed around the eNodeB. In the current downlink subframe the eNodeB sends M_k PDUs to the k-th MG and each of them is split in N_k PRBs. These PRBs are mapped in the $O \times S$ matrix and must satisfy the following power constraint:

$$\sum_{i=1}^{O} P_{i,j} \leq \hat{P}, \quad j = 1, \ldots, S \tag{1}$$

where $P_{i,j}$ is the transmission power assigned to the (i, j)-th PRB and $\hat{P}$ is the overall transmission power available at eNodeB for multicast transmissions.

Let $\overline{P} = \frac{\hat{P}}{O}$ be the *reference transmission power level*, (1) can be rewritten as follows:

$$\sum_{i=1}^{O} g_{i,j}[b, t]\; x_{b,t} \leq O, \quad j = 1, \ldots, S \tag{2}$$
$$b = 1, \ldots, K, \;\; t = 1, \ldots, M_k$$

where $g_{i,j}[b, t]$ is 1 whenever the (i, j)-th PRB holds data belonging to the t-th PDU directed to the b-th MG or 0 otherwise. $x_{b,t}$ takes not negative real values and represents the Power Scaling Factor (PSF) assigned to the t-th PDU directed to the b-th MG (i.e., $P_{i,j} = x_{b,t}\; \overline{P}$). The power adaptation scheme that will be proposed in next section results in an optimization strategy of the PSFs.

In this paper we assume that each PRB is characterized by the same capacity (that can be defined as the maximum amount of data that it can hold) hence, the downlink traffic flows rely always on the same modulation order. Moreover, we assume also that:

- The number of PRBs per PDU is considered fixed and the same for all the MGs, i.e., $N_k = N$ $(k = 1, \ldots, K)$;
- The signal passes through a slow varying Rayleigh fading channel [17] whose realization is assumed to be constant for all the PRBs belonging to the same PDU in a downlink subframe. The fading coefficients are assumed to be known at the eNodeB side.

B. Network Coding communication scheme

The eNodeB transmits a traffic flow to each MG (i.e., all the UEs belonging to a MG will receive the same flow). All communication flows relay on a RLNC scheme implemented at the MAC layer (we will refer to it with MAC-RLNC).

Each message directed to a specific MG consists of a stream, called *generation*, of l PDUs (h symbols long). In the rest of the paper we will equivalently refer to l as "generation size" or "generation length". The i-th PDU $\mathbf{s}_i$ (with $i = 1, \ldots, l$) can be represented as a column vector of symbols h. The matrix $\mathbf{M} = [\mathbf{s}_1; \mathbf{s}_2; \ldots; \mathbf{s}_l]$, of dimension $h \times l$ represents the *message*.

Following the RLNC scheme, the eNodeB progressively sends linear combinations of the PDUs composing the message $\mathbf{M}$. For a correct reception, each UE (belonging to a given MG) must receive l linearly independent combinations of the PDUs composing the message. However, with RLNC all the coding vectors could not be independent. From the main theorem of NC [10] and the theory of RLNC [18] follows that $l + e$ coding vectors must be randomly generated to define a valid RLNC scheme: where e is the amount of the redundant coding vectors. The resulting i-th coded PDU $\mathbf{r}_i$, with $i = 1, \ldots, l + e$ is given as follows:

$$\mathbf{r}_i = \mathbf{M} \times \mathbf{c}_i$$

where $\mathbf{c}_i$ is a randomly generated coding column vector l symbols long. In particular we assume that all the coding operations are performed over a large enough finite field $\mathcal{F}_q$ of size q. For this reason each coding vector can take q^l possible values.

Each coded PDU $\mathbf{r}_i$ is transmitted to the corresponding MG. Both the coding vectors and the generation size are supposed known at the receiver side. Only the linearly independent coded PDUs are considered in order to recover the transmitted message.

Let $\hat{\mathbf{r}}_i$ be the i-th received coded PDU and $\hat{\mathbf{c}}_i$[1] the corresponding coding vector, when the number of linearly independent coded PDUs is equal to l, the original message can be decoded as follows:

$$\mathbf{M} = \hat{\mathbf{R}} \times \hat{\mathbf{C}}^{-1}$$

where $\hat{\mathbf{R}} = [\hat{\mathbf{r}}_1; \hat{\mathbf{r}}_2; \ldots; \hat{\mathbf{r}}_l]$ and $\hat{\mathbf{C}} = [\hat{\mathbf{c}}_1; \hat{\mathbf{c}}_2; \ldots; \hat{\mathbf{c}}_l]$ are matrices of dimension $h \times l$ and $l \times l$, respectively. The i-th column of $\hat{\mathbf{R}}$ is the i-th received linearly independent coded PDU $\hat{\mathbf{r}}_i = [\hat{r}_1, \ldots, \hat{r}_h]^T$ and the i-th column of $\hat{\mathbf{C}}$ is the corresponding coding vector $\hat{\mathbf{c}}_i = [\hat{c}_1, \ldots, \hat{c}_l]^T$.

When an UE is able to decode the message (i.e, has collected at least l linearly independent PDUs) it sends an acknowledgement message (over a fully reliable channel) to the eNodeB. The base station begins the transmission of the next generation only when it has collected all the acknowledgement messages coming from all the UEs belonging to the same MG. For the sake of analysis we assume that an UE can be member of just one MG.

III. A Convex Power Adaptation Strategy for RLNC Schemes

This section deals with the description of a novel downlink power adaptation strategy in LTE networks relying on a MAC-RLNC communication scheme. The proposed method maximizes the system throughput and guarantees the fairness among the users. In particular it allows to increase network performance in terms of throughput and delivery delay for the UEs suffering of bad propagation conditions without loss of performance for what concerns the UEs under better propagation conditions.

We propose a non-linear model for the power adaptation and a way to efficiently solve it using a concave envelope [19]. In particular, Boyd *et al.* show in [20], [21] that for problems with up to a hundred variables, convex problems can be solved in the microseconds range.

As explained in Sec. II-A, the PRB belonging to the t-th PDU directed to the b-th MG is transmitted with a power $x_{b,t}\ \overline{P}$. At the end of reception each UE receives a signal power affected by pathloss, depending on its distance from the eNodeB and the channel fading. For each MG we consider the received Signal to Noise Ratio (SNR) of the UE (belonging to it) in the worst position (i.e., characterized by the worst propagation conditions); we will refer to it as the reference UE (rUE) Hence, for the t-th PDU of the b-th MG we have:

$$SNR_{b,t} = \alpha_{b,t}\ \overline{SNR}_{b,t}\ x_{b,t} \tag{3}$$

where $\overline{SNR}_{b,t}$ is the received SNR when the transmitted power is equal to the reference power $\overline{P}$ taking into account the pathloss. We assume Rayleigh distributed channel coefficients whose squared magnitudes ($\alpha_{b,t}$) are χ^2 distributed (with two degrees of freedom) [17].

The Power Adaptation Model (PAM) can be formalized as follows:

$$\max_{x_{b,t}} \left(\sum_{b=1}^{K} \sum_{t=1}^{M_b} w_{b,t}\ Pc\Big(\alpha_{b,t}\ \overline{SNR}_{b,t}\ x_{b,t}\Big) \right) \tag{4}$$

subject to (2). Where $Pc(\,\cdot\,)$ is the probability of correct reception of a PDU and $w_{b,t}$ is the *Power Scaling Weight* (PSW) introduced to take into account the underlying MAC-RLNC communication scheme: the eNodeB keeps track of the number (j) of coded PDUs actually transmitted to each MG in a generation (of size l). $w_{b,t}$ relative to the t-th PDU directed to the b-th MG follows this rule:

$$w_{b,t} = \begin{cases} 1 & \text{if } 0 \leq j < \lceil l/2 \rceil \\ \frac{2(c-1)}{l} j + 2 - c & \text{if } j \geq \lceil l/2 \rceil \end{cases} \tag{5}$$

where c is a real value parameter not less than 1 such that $w_{b,t} = c$ when $j = l$. In particular, the $w_{b,t}$ value linearly increases when j is greater than or equal to $\lceil l/2 \rceil$. For these reasons the objective function (4) is a maximization of the weighted sum of probability of correct reception relative to all the PDUs transmitted by the eNodeB (during a downlink phase). Given an instance of the PAM and a modulation scheme, it can be defined a concave envelope of $Pc(SNR)$ function[2] as follows:

$$\hat{P}c(SNR) = \begin{cases} m\ SNR & \text{if } 0 \leq SNR \leq Z \\ Pc(SNR) & \text{if } SNR > Z \end{cases} \tag{6}$$

where Z represents the value of SNR such that the straight line with m as slope (passing through the origin) is tangent at the point $(Z, Pc(Z))$. For these reasons Z is the real root of the following equation:

$$Pc(SNR) - SNR\ \frac{d}{dSNR}\Big(Pc(SNR)\Big) = 0 \tag{7}$$

Hence, m is:

$$m = \frac{Pc(Z)}{Z} \tag{8}$$

In the presence of the Quadrature Phase-Shift Keying (QPSK) modulation scheme can be provided the following approximation to the $Pc(SNR)$ function (valid for high values of SNR and when the Gray coding is adopted [17]):

$$Pc(SNR) = \left[1 - \frac{1}{2} erfc\Bigg(\sqrt{\frac{1}{2}\ SNR}\Bigg)\right]^f \tag{9}$$

where f is the PDU size (expressed in bits). By (7), (8) and (9) follows that:

- for $f = 21$ Bytes we have $Z = 9.2561$ and $m = 0.0886$;
- instead with $f = 42$ Bytes we have $Z = 10.7976$ and $m = 0.0780$.

[1]In order to explain the RLNC procedure we omit here the channel effects on the received PDU.

[2]The $Pc(\,\cdot\,)$ function is supposed differentiable.

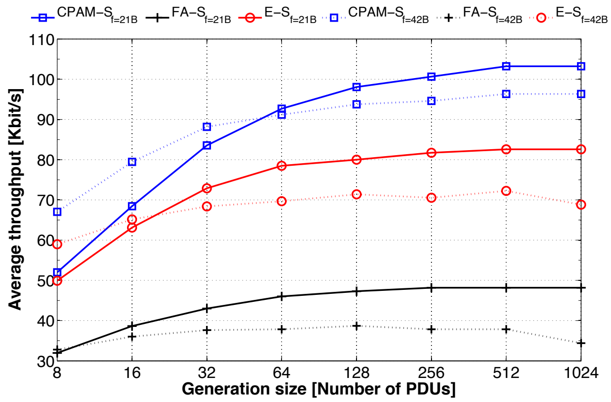

Fig. 1. The receiving throughput of MG holding the rUE experiencing the worst propagation conditions for PDUs of 21 Bytes and 42 Bytes.

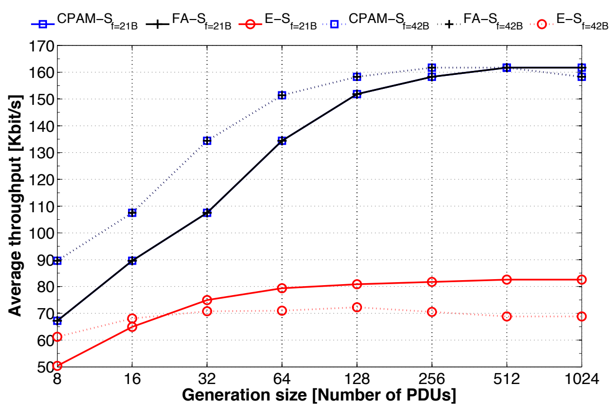

Fig. 2. The receiving throughput of MG holding the rUE experiencing the best propagation conditions for PDUs of 21 Bytes and 42 Bytes.

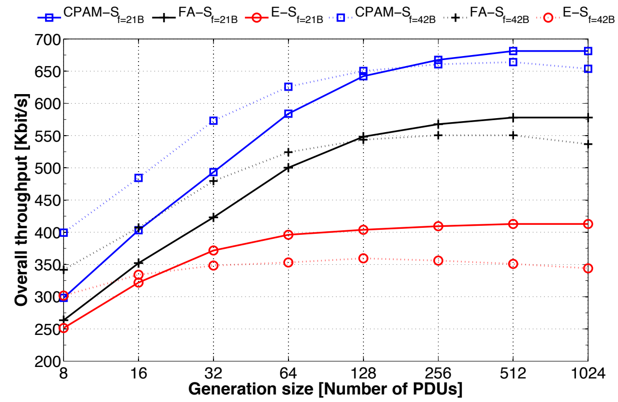

Fig. 3. Overall system throughput for different generation sizes and for PDUs of 21 Bytes and 42 Bytes.

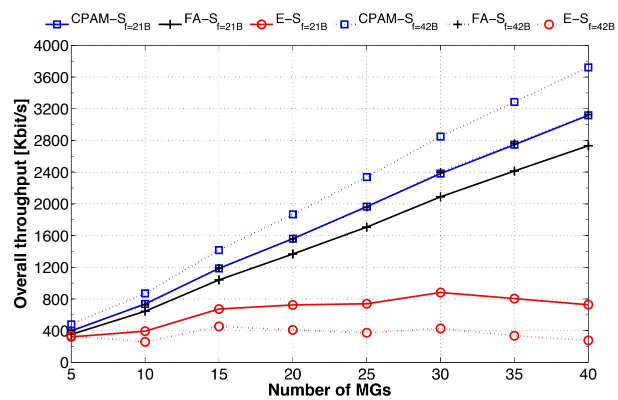

Fig. 4. Overall system throughput for networks composed by different number of MGs, using a fixed generation size of 16 PDUs (21 Bytes and 42 Bytes long).

By (6), we obtain the following convex envelope of (4):

$$\min_{x_{b,t}} \left(- \sum_{b=1}^{K} \sum_{t=1}^{M_b} w_{b,t} \; \hat{P}c\Big(\alpha_{b,t} \; \overline{SNR}_{b,t} \; x_{b,t}\Big) \right) \tag{10}$$

Hence, (10) and the constraint (2) define the proposed convex optimization model for the power adaptation, called Convex PAM (CPAM). At the beginning of each radio frame the CPAM problem is exactly solved, we will refer in the rest of the paper to the obtained power adaptation scheme as CPAM-Strategy (CPAM-S).

IV. Numerical results

In this section the numerical results are presented to validate the effectiveness of the proposed CPAM-S power adaptation strategy by resorting to computer simulations [22]. A system composed by an eNodeB and a variable number of MGs $(5 \div 40)$ randomly placed all around the eNodeB has been considered. The $\overline{SNR}_{b,t}$ values have been assumed uniformly distributed between 4.5 dB and 26 dB. All the downlink communications adopt the QPSK scheme and the same generation size, different lengths it has been considered $(8 \div 1024$ PDUs). Moreover, we have assumed networks in saturation condition: the queues at the eNodeB side holding the coded PDUs directed to each MG are always full.

CPAM optimization problem it has been solved by the SNOPT 7 solver [19]. In order to validate the CPAM-S performance, accordingly to the simulated scenarios, the c parameter of (5) has been set to 10; this value comes from an extensive set of computer simulations but due to space limitations the complete analysis cannot be reported here.

It has been compared the CPAM-S performance to the following strategies:

- The Fixed Allocation Strategy (FA-S) - all the PDUs are transmitted with the same PSF;
- The Equalization Strategy (E-S) - based on scheme presented in [23], where each PSF $(\beta_{b,t})$ is firstly calculated such as $SNR_{b,t}$ (where $b = 1, \ldots, K$ and $t = 1, \ldots, M_k$) is equal to a target SNR. Then the PSFs are normalized by a factor δ in order to respect the power constraint (2):

$$x_{b,t} = \delta \; \beta_{b,t} = \frac{O}{\sum_{i=1}^{O} g_{i,j}[b,t]} \; \beta_{b,t}$$

The target SNR value has been chosen in order to guarantee a PDU error probability less than 0.35 (representing the biggest value that we can have in the considered scenarios using CPAM-S).

Moreover, in order to make a fair comparison, the instantaneous transmitted power is always the same in a given network scenario regardless to the power adaptation strategy.

Fig. 1 shows the receiving throughput of each UE belonging to the MG holding the rUE experiencing the worst propagation conditions in a network composed by five MGs. We can note that the CPAM-S performance is always better than the E-S and FA-S (using PDUs of 21 Bytes or 42 Bytes). On the other hand, Fig. 2 shows that both CPAM-S and FA-S leads the UEs, belonging to the MG holding the rUE experiencing the best propagation conditions among the other, to receive 100% of the transmitted generations. For what concern the FA-S this behavior can be easily explained: in this case the UEs experience a very low packet error probability due to the high SNR level characterizing the downlink communications at the receiver side.

Moreover, Fig. 2 shows also that E-S is characterized by the worst performance among the other even if it leads to a fair power adaptation strategy. On the other hand, by using CPAM-S the UEs belonging to the considered MG are able to receive all the transmitted generations regardless to the chosen generation size (by exploiting the underlying MAC-RLNC communication strategy). This behavior is also confirmed by Fig. 3 showing the overall network throughput among all the UEs for different generations and PDU sizes. In particular it shows that CPAM-S is characterized by the best performance among the FA-S and E-S strategy. It is not surprising if in this case the E-S performance is significantly lower than the FA-S because: E-S allocates more power to those transmissions directed to the MGs holding the rUEs experiencing the worst propagation conditions at the expense of the other ones, unlike CPAM-S, it is not able to exploit the MAC-RLNC scheme in use. This has a significant impact on the overall downlink system throughput.

Fig. 4 shows the overall network throughput by all the UEs of a network composed by different number of MGs, adopting generations of 16 PDUs (21 Bytes or 42 Bytes long). Also in this case CPAM-S is characterized by the best performance differently from E-S; in particular this one leads to have the lowest network throughput. It should be noted that in this scenario the amount of instantaneous transmitted power is proportional to the number of MGs.

V. Conclusion

In this paper we have considered an LTE network where the UEs are clustered in MGs and each downlink communication flow relies on a MAC-RLNC scheme.

In particular, we addressed the downlink power adaptation problem providing a convex formulation by a concave envelope of the packet correct reception probability function. This ensures to find always an optimal solution to this problem with affordable computing efforts and in real time. Moreover, the proposed convex power adaptation model results in a maximization of a weighted sum of the packet correct reception probability considering each transmitted PDU. We have also shown how the weights are function of the number of coded PDUs actually transmitted to a MG exploiting the underlying MAC-RLNC communication strategy. The performance of the proposed power adaptation scheme has been compared with other alternatives, this showed a significant gain for what concern throughput and fairness among the MGs.

References

[1] S. Sesia, I. Toufik, and M. Baker, *LTE - The UMTS Long Term Evolution: From Theory to Practice*, 1st ed. WILEY, 2009.

[2] J. Jin and B. Li, "Adaptive random network coding in WiMAX," in *Proc. of IEEE ICC 2008*, Beijing, China, May 2008, pp. 2576–2580.

[3] M. Ghaderi, D. Towsley, and J. Kurose, "Reliability gain of network coding in lossy wireless networks," in *Proc. of IEEE INFOCOM 2008*, Phoenix, AZ, USA, Apr. 2008, pp. 2171–2179.

[4] M. Nistor, D. Lucani, T. Vinhoza, R. Costa, and J. Barros, "On the delay distribution of random linear network coding," *IEEE J. Select. Areas Commun.*, vol. 29, no. 5, pp. 1084–1093, may 2011.

[5] J. Jin, B. Li, and T. Kong, "Is random network coding helpful in WiMAX?" in *Proc. of IEEE INFOCOM 2008*, Phoenix, AZ, USA, Apr. 2008, pp. 2162–2170.

[6] D. Zhang and N. Mandayam, "Resource allocation for multicast in an OFDMA network with random network coding," in *Proc. of IEEE INFOCOM 2011*, Shanghai, China, Apr. 2011, pp. 391–395.

[7] R. Ahlswede, N. Cai, S.-Y. Li, and R. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. 46, no. 4, pp. 1204–1216, Jul. 2000.

[8] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Médard, and J. Crowcroft, "XORs in the air: practical wireless network coding," *SIGCOMM Comput. Commun. Rev.*, vol. 36, pp. 243–254, Aug. 2006.

[9] S.-Y. Li, R. Yeung, and N. Cai, "Linear network coding," *IEEE Trans. Inf. Theory*, vol. 49, no. 2, pp. 371–381, Feb. 2003.

[10] C. Fragouli and E. Soljanin, *Network Coding Fundamentals*. Hanover, MA, USA: Now Publishers Inc., 2007.

[11] P. Vingelmann, F. Fitzek, M. Pedersen, J. Heide, and H. Charaf, "Synchronized multimedia streaming on the iphone platform with network coding," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 126–132, Jun. 2011.

[12] J. Heide, M. Pedersen, F. Fitzek, and T. Larsen, "Network coding for mobile devices - systematic binary random rateless codes," in *Proc. of IEEE ICC Workshops 2009*, Dresden, Germany, Jun. 2009, pp. 1–6.

[13] J. Mller, M. Pedersen, F. Fitzek, and T. Larsen, "Cautious view on network coding - from theory to practice," *Journal of Communications and Networks (JCN)*, vol. 10, no. 4, pp. 403–411, 2008.

[14] I. Wong and B. Evans, "Optimal downlink ofdma resource allocation with linear complexity to maximize ergodic rates," *IEEE Trans. on Wireless Commun.*, vol. 7, no. 3, pp. 962–971, march 2008.

[15] A. Simonsson and A. Furuskar, "Uplink power control in LTE - overview and performance, subtitle: Principles and benefits of utilizing rather than compensating for sinr variations," in *Proc. of IEEE VTC 2008*, Calgary, Alberta, Sep. 2008, pp. 1–5.

[16] J. Gozalvez, "Long-term evolution: 1 Gb/s and beyond," *IEEE Veh. Technol. Mag.*, vol. 5, no. 3, pp. 7–13, Sep. 2010.

[17] J. Barry, E. Lee, and D. Messerschmitt, *Digital communication*. Kluwer Academic Publishers, 2004.

[18] P. Chou and Y. Wu, "Network coding for the internet and wireless networks," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 77–85, Sep. 2007.

[19] P. E. Gill, W. Murray, Michael, and A. Saunders, "SNOPT: An SQP algorithm for large-scale constrained optimization," *SIAM Journal on Optimization*, vol. 12, pp. 979–1006, 1997.

[20] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge University Press, 2004.

[21] J. Mattingley and S. Boyd, "Automatic Code Generation for Real-Time Convex Optimization," in *Convex Optimization in Signal Processing and Communications*, D. P. Palomar and Y. C. Eldar, Eds. Cambridge University Press, 2010, pp. 1–41.

[22] OMNeT++ Community Site. [Online]. Available: http://www.omnetpp.org

[23] C. Y. Wong, R. Cheng, K. Lataief, and R. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Select. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.