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RESEARCH ARTICLE

An Efficient HARQ Scheme For Applications in Multicast Communication Systems

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ABSTRACT

This paper deals with the performance evaluation and optimization of an efficient Hybrid Automatic Repeat Request (HARQ) scheme suitable for applications delivered over lossy multicast communication channels. In particular, differently from previously investigated strategies, this paper proposes a Modified HARQ scheme based on the Symbol Combining principle (MHARQ-SC) where multiple copies of a same packet are consecutively transmitted at each transmission opportunity. By considering as performance metrics the mean packet delivery delay and energy consumption per information packet, this paper presents suitable performance evaluation and optimization strategies tailored for multicast communications. For the sake of comparisons, it has been also analysed, under the same operational conditions, the performance of different HARQ schemes optimized for multicast communications. Numerical results have been provided in order to validate the proposed performance evaluation and optimization approaches in the case of the MHARQ-SC scheme. An important result devised here is that the reported analytical results clearly highlights the performance gain of the proposed strategy in comparison with all the other considered alternatives. Copyright © 2013 John Wiley & Sons, Ltd.

KEYWORDS

HARQ scheme; Lossy Multicast Wireless Networks; Multicast Delay Optimization

1. INTRODUCTION

Multicast communications are gaining momentum in wireless communications since they make possible the delivery of services to multiple users located within the coverage area of the same access node. Differently from unicast communications, multicast flows can transmit the same message to several users by (virtually) just one transmission attempt. Hence, multicast services can reduce the spectrum demand and energy consumption. Multicast services include video applications, group text messaging and specific messaging services to handle emergency situations (in geocasting fashion).

On the other hand, it is well known in the literature [1–8] that the communication delay and transmission energy associated to reliable multicast communications decrease as the number of users increases. In particular, the mean time required to successfully transmit an information packet to all the members of a Multicast Group (MG) increases with the MG size; hence, the overall energy increasing related to the reliable transmission of an information packet increases as well. It can be noticed that usually the bottleneck of the overall system performance is the node experiencing the worst channel propagation conditions. Reliable communications are normally guaranteed by the adoption of Hybrid Automatic Repeat Request (HARQ) schemes. At the light of this, the paper deals with HARQ schemes, as foreseen in 3G HSDPA (High Speed Downlink Packet Access) standard [9]. In particular, these aim to reliably transmit downlink communications flows characterized by a rate higher than that of the 3G UMTS cellular system [2].

The HARQ scheme has been first considered for increasing data reliability in unicast communications [10, 11]. In particular, Lin *et al.* proposed in [10] a HARQ technique that iteratively broadcasts fixed-length packets comprised of a couple of subpackets of equal length. In this way, each subpacket can be used to deliver new information elements or for increasing the amount of redundancy related to a packet previously transmitted.

On the other hand, several papers proposed the use of HARQ schemes in multicast communication systems [1, 8, 12–14]. In particular, Wang *et al.* [15] proposed several HARQ strategies. Among them, the authors proposed schemes exploiting the repetition time diversity; where the receiver recover transmitted information packets by combining (according to the maximum ratio combining) a fixed number of copies previously received (with errors). In addition, Kim *et al.* proposed in [12] a HARQ error control strategy for multicast communication based on the Symbol Combining (SC) approach, originally proposed by Chase [16]. The SC approach requires that each receiving node stores all the copies of the same information packet (even those received with errors) and, upon the reception of a packet, it is soft-combined with the previous ones. Hence, the transmission reliably is ensured, increasing the system performance. In [12] an extensive performance comparison with different HARQ alternatives already presented in the literature is provided to clearly highlight the better behaviour of the proposed approach, referred in the following as HARQ-SC scheme.

Differently from [12], this paper deals with a Modified HARQ-SC scheme, hereafter named MHARQ-SC, relaying on the *continuous* transmission (or retransmission) of a specified number of copies (denoted as m) of the *same* packet at each transmission opportunity. At each receiving node, all the received copies of the same packet are combined according to the SC principle. It follows that the implementation complexity of the proposed MHARQ-SC is the same as for the classical HARQ-SC scheme.

In order to derive the performance bounds for the MHARQ-SC scheme a theoretical framework based on the Absorbing Markov Chains (AMC) theory [17] is proposed hereinafter. We will consider in the analysis both AWGN and frequency-non-selective slow Rayleigh faded propagation conditions. Moreover, an efficient optimization procedure of the MHARQ-SC scheme is also outlined in the paper. The accuracy of the proposed performance evaluation and optimization methods will be validated by resorting to computer simulations. Finally, we will show that the optimized MHARQ-SC scheme outperforms the rate optimized HARQ-SC alternative proposed in [12].

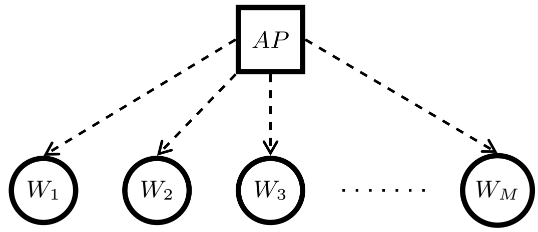

Figure 1. Multicast network model.

The remaining part of this paper is organized as follows. In Section 2 we introduce and analyse the MHARQ-SC scheme for multicast communications in the cases of AWGN and (frequency-non selective slowly) faded communication channels. A suitable optimization method is also proposed and analysed in this Section. Numerical results and performance comparisons are provided in Section 3. Finally, in Section 4 the conclusions are drawn.

2. MHARQ-SC APPROACH IN MULTICAST COMMUNICATIONS

This Section deals with the performance evaluation and optimization of the MHARQ-SC scheme for multicast communication systems. The accuracy of the proposed approach will be validated by comparing analytical predictions with simulation results. As stated before, the MHARQ-SC scheme resorts to the SC principle previously considered [11, 16–18] in order to increase the overall delivery performance, in particular under high error rate conditions. It will be demonstrated here that the continuous transmission of an optimized number m of copies of a same packet at each transmission opportunity allows the MHARQ-SC scheme to behave better than the HARQ-SC alternative. In particular, the impact of the round-trip delay is reduced, i.e., the time needed to send a packet from the transmitting node to the MG and to receive the successful feedback from all the members of the MG.

The functional model of the system under investigation is sketched in Fig. 1. In this case, a single source

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node, namely the Access Point (AP), transmits the same communication flow to the set $\mathcal{W} = \{W_1, W_2, \ldots, W_M\}$ of M nodes forming the MG. Each link from the AP to a generic node is modelled as an independent lossy channel. In such scenario, the simplest retransmission scheme consists in retransmitting a new set of m copies of a same packet till all the nodes belonging to the MG receive that packet without errors* [8]. However, this solution is not efficient because it cannot take into account that some nodes could have successfully received the informative packet during previous transmission attempts. As a consequence, we refer here to a more efficient scheme where the AP keeps track of the received acknowledgment messages individually sent out by each node. Hence, according to this scheme, the AP continuously retransmits a set of m copies of a same packet if there is *almost* one node within the MG waiting for that packet; the other nodes instead ignore successive error free receptions of the same packet. In particular, any node of the MG recover the information packet by using the procedure below:

- The m copies of the same packet are combined symbol-by-symbol according to the SC principle;
- The SC detection is performed symbol-by-symbol as outlined in the Appendix A;
- Assuming that we adopt an ideal error detection code (i.e., a code able to detect any error pattern), in the case of the information packet has been successfully received, a positive acknowledgement message (ACK) is sent back to the AP, a Negative ACK (NACK), otherwise.

The algorithmic description of the MHARQ-SC scheme, from the point of view of the AP, is illustrated in Fig. 2. Note that by assuming m equal to 1 the same figure also illustrates the mode of operation of the classical HARQ-SC alternative. In particular, the classical HARQ-SC scheme consists in a packet transmission procedure where information packets are soft-combined by each receiving node (on a packet transmission basis). In this case, the SC detection process is reset as soon as the information packet has been successfully received (i.e., a SC detection is successfully performed). On the other hand, in the case of the MHARQ-SC approach, the receiving node performs the SC detection on the set of m copies of the same information packet. If the detection process fails, a new process start as soon as a new set of m copies are received.

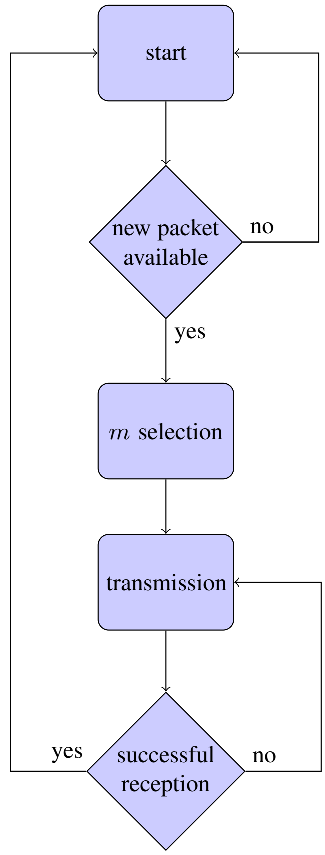

Figure 2. Algorithmic description of the MHARQ-SC scheme from the point of view of the AP.

Finally, the theoretical analysis presented in the paper assumes saturation conditions (i.e., the AP always have information packets to transmit).

We will show in the rest of this paper that the performance of the MHARQ-SC scheme are dependent on m. So that an analytical approach to derive optimum m values (maximizing the performance gain in comparison with the HARQ-SC alternative) is presented. We will mainly focus on the minimization of the mean multicast delivery delay (on a information packet basis) defined as the time elapsed from the beginning of the first transmission attempt of the set of m copies of the same packet to the time instant at which all the nodes of the MG have notified to the AP the successful reception of that packet, normalized with respect to the single packet transmission time τ.

*Without any loss of generality we assumed that the acknowledgement messages are transmitted on a fully reliable channel.

Finally, we will assume that: (i) the communication channels are iid with an AWGN or frequency-non selective slow Rayleigh faded propagation conditions and, (ii) each information flow uses the Binary Phase Shift Keying (BPSK) modulation[†]. Moreover, propagation conditions will be considered statically independent across the transmission of the m copies of the same information packet.

2.1. Performance Evaluation

The performance behaviour of the MHARQ-SC scheme can be efficiently inspected by the Absorbing Markov Chain (AMC) theory [17] (Chapter 3, page 43). Let us consider the process modelling the error-free reception of an information packet from all the nodes belonging to the MG. We start our analysis by considering a scenario where all the communication channels have same propagation conditions.

Let s_i (for $i = 0, \dots, M$) be the state of the process, defined as the number of nodes of the MG that have successful received the packet of interest. Hence, if at the end of a next attempt the number of nodes that have got an error-free copy of the information packet passes from i to j (for $j \geq i$), the process moves from the state s_i to s_j. According to this, s_M is the *absorbing* state of the process [17], i.e., the process enters this state if all the nodes of the MG received an error-free copy of the original packet. So that, once the process reaches the absorbing state, it cannot be left.

It is easy to note that the considered process always starts from the state s_0 and ends as soon as it enters the absorbing state s_M. According to standard AMC theory definitions, all the states $\{s_0, \dots, s_{M-1}\}$ are *transient*, meaning that once the process leaves one of them it can no longer reenter it.

[†]Note that the derived results are quite general and they can be easily extended to different modulation schemes.

Let $p_{i,j}$ be the probability that a transition occurs from the state s_i to s_j, defined, for $j \geq i$, as:

$$p_{i,j} = \binom{M-i}{M-j} P_B(m)^{M-j} \Big[1 - P_B(m)\Big]^{j-i} \quad (2)$$

where $P_B(m)$ is the packet error probability at each receiving end and depends upon m as justified in the Appendix A.

Let $\mathbf{P}$ be the state transition probabilities matrix with the (i,j)-th entry, $p_{i,j}$, given by (2). In particular, $\mathbf{P}$ can be expressed as reported in (1).

Moreover, we have from the standard AMC theory [17] that the transition matrix $\mathbf{P}$ can be expressed in its canonical form as follows:

$$\mathbf{P} = \left[\begin{array}{c|c} \mathbf{Q} & \mathbf{R} \\ \hline \mathbf{0} & 1 \end{array}\right] \quad (3)$$

where:

1. $\mathbf{Q}$ is the $M \times M$ transition matrix modeling the process as long as it involves only transient states, defined as:

$$\mathbf{Q} \doteq \begin{bmatrix} P_B(m)^M & \cdots & \binom{M}{1} P_B(m) \Big[1 - P_B(m)\Big]^{M-1} \\ 0 & \cdots & \binom{M-1}{1} P_B(m) \Big[1 - P_B(m)\Big]^{M-2} \\ \vdots & \vdots & \vdots \\ 0 & \cdots & P_B(m) \end{bmatrix} \quad (4)$$

2. $\mathbf{R}$ is a M-dimensional column vector listing the transition probabilities of the process whenever it starts from a transient state and enters the absorbing one, defined as:

$$\mathbf{R} \doteq \begin{bmatrix} \Big[1 - P_B(m)\Big]^M \\ \Big[1 - P_B(m)\Big]^{M-1} \\ \vdots \\ 1 - P_B(m) \end{bmatrix} \quad (5)$$

$$\mathbf{P} \doteq \begin{bmatrix} P_B(m)^M & \binom{M}{M-1} P_B(m)^{M-1} \Big[1 - P_B(m)\Big] & \cdots & \Big[1 - P_B(m)\Big]^M \\ 0 & P_B(m)^{M-1} & \cdots & \Big[1 - P_B(m)\Big]^{M-1} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 - P_B(m) \\ 0 & 0 & \cdots & 1 \end{bmatrix} \quad (1)$$

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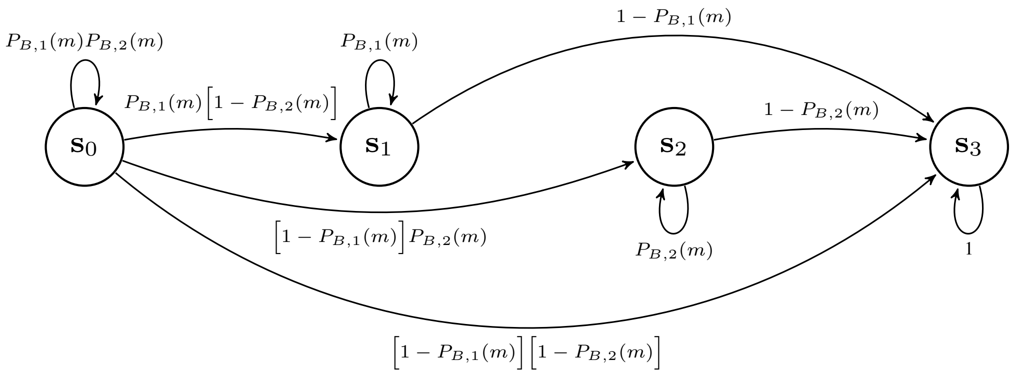

Figure 3. Absorbing Markov Chain associated to the packet transmission process for a multicast network with two receiving nodes and independent lossy channels.

3. $\mathbf{0}$ is a M-dimensional row vector with entries all equal to zero.

We are now in the position to extend the analysis outlined before to the case of *unequal* propagation conditions for each communication channel. Differently from before, the i-th *state* of the packet reception process is now defined as a M-ary column vector $\mathbf{s}_i$ (with $i = 0, \ldots, 2^M - 1$), whose t-th entry $\mathbf{s}_i[t]$ (for $i = 0, \ldots, 2^M - 1$ and $t = 1, \ldots, M$) is equal to 1 if node W_t has correctly received the information packet or 0, otherwise. AS a consequence, the absorbing state is represented by the vector $\mathbf{s}_{2^M-1}$ with all entries equal to one.

Also in this case, the packet transmission process begins from the state $\mathbf{s}_0$ (whose entries are all equal to 0) and ends as soon as it enters the absorbing state $\mathbf{s}_{2^M-1}$. As before, it is easily to note that all the states $\mathbf{s}_i$ (with $i = 0, \ldots, 2^M - 2$) are transient.

Let us define the operator $\succeq$: the relation $\mathbf{s}_j \succeq \mathbf{s}_i$ holds if $\mathbf{s}_j[t] \geq \mathbf{s}_i[t]$ $\forall t = 1, \ldots, M$. Let $P_{B,i}(m)$ (for $i = 1, \ldots, M$) be the packet error probability at the i-th node side, i.e., the probability that the W_i node cannot recover the information packet after processing the m received copies according to the SC technique. As reported in the in the Appendix A, Eqs. (20) and (26) define $P_{B,i}(m)$ (for $i = 1, \ldots, M$) in the case of AWGN and frequency-non selective slow Rayleigh faded channel, respectively.

Hence, for $\mathbf{s}_i = \mathbf{s}_j = \mathbf{s}_{2^M-1}$, the (i, j)-th entry of the transition matrix $\mathbf{P}$ is equal to one. Otherwise, if $\mathbf{s}_j \succeq \mathbf{s}_i$, the state transition probability $p_{i,j}$ is defined as:

$$p_{i,j} = \prod_{\substack{t=1,\ldots,M \text{ t.c.} \\ \mathbf{s}_i[t]=\mathbf{s}_j[t]=0}} P_{B,t}(m) \prod_{\substack{t=1,\ldots,M \text{ t.c.} \\ \mathbf{s}_j[t]>\mathbf{s}_i[t]}} \left[1 - P_{B,t}(m)\right] \quad (6)$$

As an example, the state diagram of an AMC modelling the packet error-free reception for a multicast group formed by two nodes (under the different propagation conditions assumption) is shown in Fig. 3.

From (6), the transition probability matrix $\mathbf{P}$ can be again given in the canonical form (3). However, in this case we have:

1. $\mathbf{Q}$ is a $(2^M - 1) \times (2^M - 1)$ defined as:

$$\mathbf{Q} = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,2^M-1} \\ p_{1,0} & p_{1,1} & \cdots & p_{1,2^M-1} \\ \vdots & \vdots & \vdots & \vdots \\ p_{2^M-1,0} & p_{2^M-1,1} & \cdots & p_{2^M-1,2^M-1} \end{bmatrix} \quad (7)$$

2. $\mathbf{R}$ is a $(2^M - 1)$-dimensional column vector defines as:

$$\mathbf{R} = \begin{bmatrix} p_{0,2^M} \\ p_{1,2^M} \\ \vdots \\ p_{2^M-1,2^M} \end{bmatrix} \quad (8)$$

3. $\mathbf{0}$ is a $2^M - 1$ dimensional row vector with all entries equal to zero.

In oder to derive a closed form solution for the the mean multicast delivery delay, it is useful to provide the following definition [17] (Chapter 3, page 46):

Definition 1
Let us consider the matrix $\mathbf{Q}$ (3); we define the fundamental matrix of the AMC as $\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$, where $\mathbf{I}$ is the identity matrix having $M \times M$ dimension in the case of equal propagation conditions or $(2^M - 1) \times (2^M - 1)$ dimension in the case of unequal propagation conditions.

Moreover, from standard AMC theory, we have that the following relation holds [17] (Chapter 3, page 46):

$$\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1} = \sum_{k=0}^{\infty} \mathbf{Q}^k \tag{9}$$

where the (i, j)-th component of the matrix $\mathbf{Q}^k$ is the probability that the process reaches the j-th transient state from the j-th one in exactly k steps (i.e., after that the AP has complete k transmissions of the set of m copies of the same information packet).

From (9) we have that the (i, j)-th element of the fundamental matrix N is the mean value of the total number of times that the process, started from the i-th state, reaches the j-th state[‡]. Hence, the mean number of times that the AP has to transmit m copies of the same information packet before all the users receive the original packet (i.e., before that the process enters the absorbing state) can be expressed as

$$\zeta(m) = \sum_{l} \mathbf{N}[1, l] \tag{10}$$

where the term $\mathbf{N}[1, l]$ is the $(1, l)$-th element of the $\mathbf{N}$ matrix. Hence, the term $\zeta(m)$ is equal to the sum of all the elements forming the first row of the matrix $\mathbf{N}$.

2.2. Optimization Procedure

From (10), we have that the mean multicast delivery delay $\delta(m)$ of an informative packet (normalized with respect to τ) is:

$$\delta(m) = (m + s)\,\zeta(m) \tag{11}$$

where s denotes the round-trip delay normalized with respect to τ (i.e., the normalized time elapsed from the successful reception of the information packet to the reception of the ACK by the AP).

[‡]Note that both the i-th and j-th states are transient.

Let L and E_b be the packet length (expressed in bits) and the energy per bit, respectively. The mean value (normalized with respect to LE_b) of the mean energy, $\epsilon(m)$, required to successful deliver an information packet can be expresses as follows:

$$\epsilon(m) = m\,\zeta(m) \tag{12}$$

From (11) and (12) we note that the mean energy is minimized once $\epsilon(m)$ is minimized with respect to m. Hence, for the sake of simplicity, we focus our analysis only on the minimization of $\delta(m)$ with respect to m. As a consequence, the optimal values for m have been assumed as the solutions of the following optimization problem:

$$\text{(P1)} \qquad \text{minimize} \quad \delta(m) \tag{13}$$

$$\text{subject to} \quad \hat{m} \in \mathbb{N} \tag{14}$$

Unfortunately, P1 is an integer nonlinear optimization problem. This means that it is hard to the best of our knowledge to derive its solution in a closed formulation. As a consequence, we have resorted to the Mesh Adaptive Direct Search (MADS) algorithm [19] that allows to solve non-differentiable and nonlinear problems (possibly having integer optimization variables). In particular, in this paper we have considered the implementation of the MADS algorithm based on the NOMAD solver [20].

3. NUMERICAL RESULTS

This Section presents numerical results concerning the performance analysis of the proposed MHARQ-SC scheme, while providing comparisons with different HARQ alternatives in order to clearly highlight the advantages of the proposed solution. In particular, the classical HARQ-SC scheme (Sec. 2) and the optimized HARQ approach proposed by Kim *et al.* [12] have been considered. The analytical predictions derived according to the performance evaluation method outlined in Sec. 2 will be also validated by comparisons with simulation results.

The cases of an AWGN and non-selective slow Rayleigh faded propagation conditions have been assumed within two different scenarios detailed below:

- *Scenario 1* - All the links connecting the AP to the receiving nodes have the same propagation

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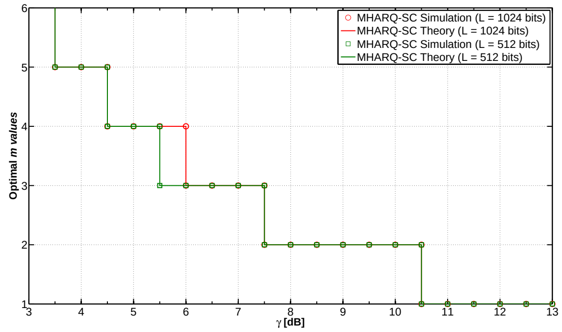

Figure 4. Optimal values of m as function of γ [dB] in the case of AWGN propagation conditions (*Scenario 1*).

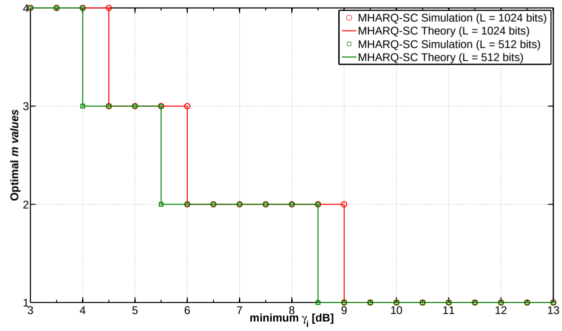

Figure 5. Optimal values of m as function of the minimum values of γ_i [dB] in the case of AWGN propagation conditions (*Scenario 2*).

conditions. This means that the Signal-to-Noise Ratio (SNR) γ_i and the mean SNR $\bar{\gamma}_i$, at the i-th receiving node (for $i = 1, \ldots, M$) are equal for all the nodes to a value γ and $\bar{\gamma}$ (i.e., $\gamma_i = \gamma$ and $\bar{\gamma}_i = \bar{\gamma}$), respectively. Both γ and $\bar{\gamma}$ take the values spanning the interval $[3, 13]$ dB;

- *Scenario 2* - Different propagation conditions with parameters γ_i and $\bar{\gamma}_i$ defined (for $i = 1, \ldots, M$) as

$$\gamma_i = \gamma_1 - (i-1)\eta, \quad i = 1, \ldots, M \qquad (15)$$

$$\bar{\gamma}_i = \bar{\gamma}_1 - (i-1)\bar{\eta}, \quad i = 1, \ldots, M \qquad (16)$$

where the values of η and $\bar{\eta}$ have been set to 0.45 dB. Hence, in this scenario the node W_1 is characterized by the best propagation conditions while node W_M the worst ones.

Moreover, we have assumed for both considered scenarios:

- a multicast set composed by $M = 30$ nodes;
- the length L of each information packet equal to 512 or 1024 bits;
- the normalized round-trip time, s, defined in Sec. 2 equal to 5 or 10.

We start our analysis by providing numerical results concerning the optimization procedure for the proposed MHARQ-SC scheme outlined in Sec. 2 according to the problem (P1). The obtained results are shown in Figs. 4 and 5 as function of γ and $\bar{\gamma}$, respectively, under the assumption of AWGN propagation conditions. In these figures the optimal m values derived as solutions of the problem P1 are compared with those ones obtained by resorting to numerical simulations in order to validate the proposed optimization procedure. A good agreement between analytical predications and simulation results is evident in the figures.

For what concerns the latter aspect, the value of m has been optimized by simulating the transmission of the same information packet and testing different values of m[§], the value minimizing (13) was considered as the best value for the m index (among the considered ones). On the other hand, we remark that the optimal value of m (representing the solution of P1) has been found by using the NOMAD solver (Sec. 2). Finally, let us consider again Figs. 4 and 5, they show that both the optimal values of m and those derived by the simulative approach are the same.

The performance in terms of the mean multicast delivery delay in the case of the optimized MHARQ-SC scheme (normalized with respect to τ) are presented in Figs. 6 and 7. The figures also compare the MHARQ-SC with the classical HARQ-SC scheme by assuming $L = 1024$ bits and $M = 64$ nodes. These figures clearly highlight a good agreement between analytical predictions and simulation results. In addition to this, a significant performance gain of the proposed MHARQ-SC scheme with respect to the other considered alternatives is pointed out.

Let us consider frequency-non selective slow Rayleigh faded propagation conditions. According to this, Figs. 4-5 and Figs. 8-9 show the obtained optimal m values. The figures also compare both analytical and simulation results; also in this case we can not that the analytical predictions match the simulation results.

[§] In particular we have considered values in the interval $[1, 30]$.

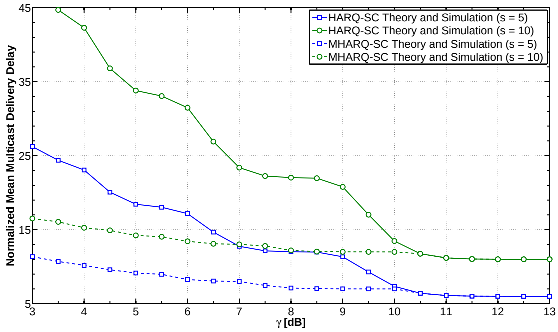

Figure 6. Normalized mean multicast delivery delay (per information packet) as function of γ [dB] in the case of AWGN propagation conditions (*Scenario 1*).

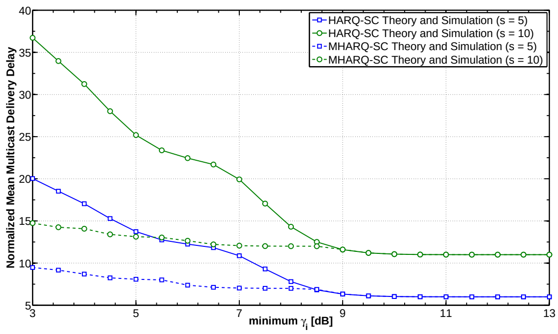

Figure 7. Normalized mean multicast delivery delay (per informative packet) as function of the minimum value of γ_i [dB] in the case of AWGN propagation conditions (*Scenario 2*).

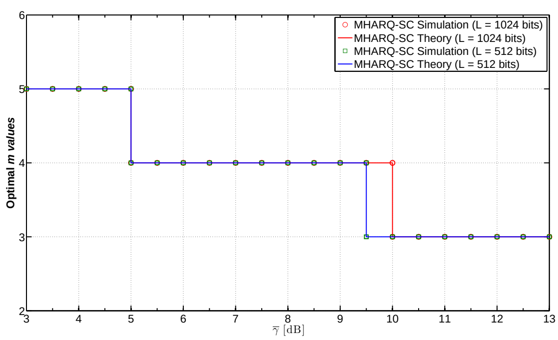

Figure 8. Optimal m values as function of $\bar{\gamma}$ [dB] in the case of (frequency non-selective slow Rayleigh) faded propagation conditions (*Scenario 1*).

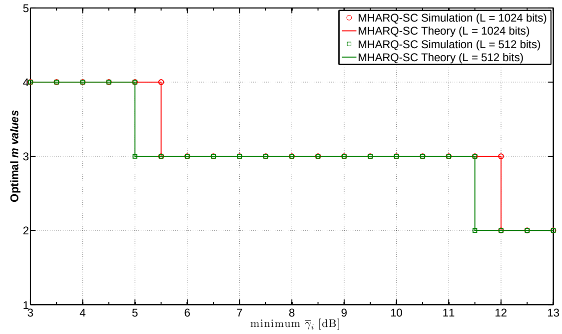

Figure 9. Optimal m values as function of the minimum value of $\bar{\gamma}_i$ [dB] in the case of (frequency non-selective slow Rayleigh) faded propagation conditions (*Scenario 2*).

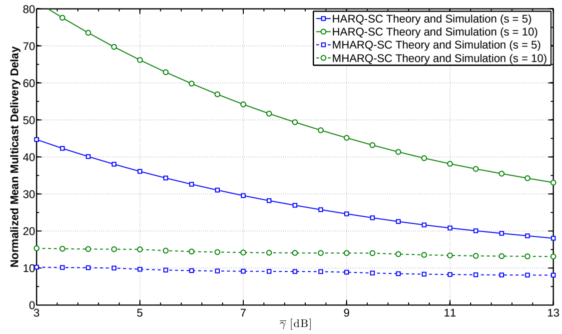

Figure 10. Normalized mean multicast delivery delay for informative packet as a function of $\bar{\gamma}$ [dB] for a frequency non-selective, slow Rayleigh fading channel (*Scenario 1*).

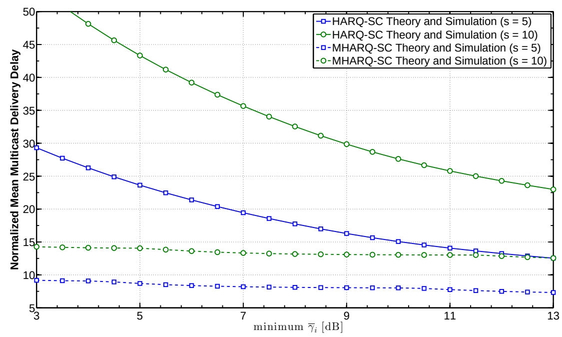

Figure 11. Normalized mean multicast delivery delay for informative packet as a function of the minimum $\bar{\gamma}_i$ [dB] for a frequency non-selective, slow Rayleigh fading channel (*Scenario 2*).

Figs. 10-11 show the mean (normalized) multicast delivery delay performance of the optimized MHARQ-SC scheme in comparison with those achieved by the classical HARQ-SC, for $L = 1024$ and M=64. Analytical predictions are once again compared with simulation results in order to validate the proposed performance evaluation and optimization approaches. These figures clearly point out the better behaviour of the optimized MHARQ-SC scheme.

Likewise, Fig. 12 compares the performance of the optimized MHARQ-SC scheme with that achieved by the rate optimized HARQ-SC scheme proposed by Kim *et al.* [12] (indicated in the figure as Opt. HARQ-SC). The results reported in the figure were derived

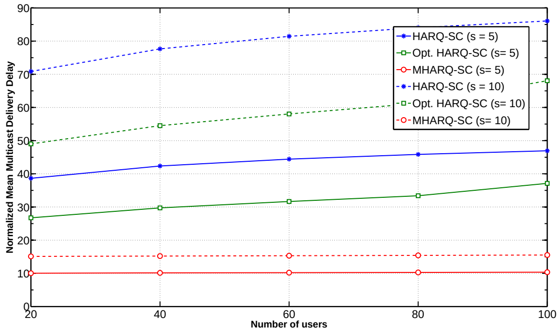

Figure 12. Normalized mean multicast delivery delay as a function of M for $\bar{\gamma}_i = 3$ dB.

by considering the Scenario 1 with $\bar{\gamma}_i = 3$ dB (for $i = 1, \ldots, M$) and (frequency-non selective slow Rayleigh) faded propagation conditions. In the same figure the performance of the classical HARQ-SC scheme are also reported for comparison purposes. In particular, we stress that in Fig. 12, the proposed MHARQ-SC clearly outperforms the other alternatives, in particular, when the number of multicast nodes increases.

Finally, we would like to point out that the improved performance of the MHARQ-SC scheme, in comparison with the classical and optimized HARQ-SC schemes, can tackle the drawback of excluding from the service all these users characterized by poor channel propagation conditions. This is of particular interest in the case of real time services where the so-called *Quality of Experience* (QoE) index plays a crucial role [21].

4. CONCLUSIONS

In this paper we have investigated the performance improvements achieved by the use of the proposed MHARQ-SC scheme in multicast wireless communication systems. The paper has also proposed a suitable optimization procedure in order to achieve the best performance for the MHARQ-SC scheme. Both the performance analysis and the optimization strategy proposed for the MHARQ-SC scheme have been based on the application of the AMC theory. Furthermore, the accuracy of the obtained analytical predictions have been validated by comparing them with simulation results in the case of AWGN and frequency-non-selective slow Rayleigh faded propagation conditions. Performance comparisons with the classical HARQ-SC scheme and optimized HARQ-SC have been also provided to highlight the performance improvement of the proposed MHARQ-SC scheme. Finally, to further validate the good behaviour of the optimized MHAR-SC scheme, we have carried out a performance comparison with an optimized HARQ-SC recently proposed in the literature. The effectiveness of the proposed optimization and analytical model is evident especially if we compare it with the optimized HARQ-SC scheme. It demonstrate significant throughput gains of 2.6-3.6 fold in the mean multicast delivery delay.

A. APPENDIX

This Appendix deals with the definition of $P_B(m)$ at a generic node of the MG. This allows us to keep the analysis general and simplify the notation considering at the same time both Scenario 1 and 2. In order to facilitate a better understood of what follows, we recall that according to the MHARQ-SC principle the AP continuously transmits m copies of a same packet at each transmission attempt.

In carrying out our analysis we considered both AWGN and frequency non-selective slowly-faded Rayleigh multipath fading channel. In the rest of this Appendix, we will refer with γ (AWGN regime) and $\bar{\gamma}$ (frequency non-selective, slow Rayleigh multipath fading regime) to the SNR and mean SNR characterizing the reception of a generic node, respectively.

A.1. AWGN Propagation Conditions

Let us assume that the receiving node performs an ideal coherent detection. The decision variable [22] for each of the L bits forming the j-th received copy (of the same packet) is:

$$z_i(j) = d_i\sqrt{E_b} + n_i(j) \quad i = 1, \ldots, L \qquad (17)$$

where d_i is a random variable taking the values $\{-1, +1\}$ with equal probability if the i-th bit is 0 or 1, respectively. The term E_b is the energy associated to each transmitted bit and $n_i(j)$ is random variable which follows a Gaussian distribution with zero mean zero and variance $N_0/2$ (where N_0 is the one-side power spectral density of the white Gaussian noise affecting the link between the AP and the generic node).

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According to the SC principle [23], we can define the following overall decision variable:

$$Z_i = \sum_{j=1}^{m} z_i(j) = m\, d_i \sqrt{E_b} + n_i \quad i = 1, \ldots, L \quad (18)$$

where it is easily to note that the term n_i results to be a Gaussian random variable with zero mean and variance equal to $m\, N_0/2$.

According to the standard decision criterion for equiprobable symbols [22], we have that the bit error probability related to the BPSK modulation is:

$$P_e(m) = Q\left(\sqrt{2\, m\, \gamma}\right) \quad (19)$$

where $\gamma = E_b/N_0$. Hence, under the assumption of an ideal detecting code,i.e., able to detect all the error configurations, the error probability related to an information packet (as function of m) is:

$$P_B(m) = 1 - \Big[1 - P_e(m)\Big]^L . \quad (20)$$

A.2. Frequency Non-Selective Slow Rayleigh Multipath Fading case

Let us assume that the communication channel connecting the AP with the receiving node is a frequency non-selective, slow Rayleigh multipath faded. Moreover, we assume that all the channel parameters are known at the AP side.

For these reasons, according to the optimal maximal ratio combiner (proposed by Brennan [23]), the overall decision variable of the i-th bit received by the generic node can be defined as follows:

$$\tilde{Z}_i = \sqrt{E_b} \sum_{j=1}^{m} \alpha_j^2 + \sum_{j=1}^{m} \alpha_j\, n_j \quad i = 1, \ldots, L \quad (21)$$

where n_j (for $j = 1, \ldots, m$) are m *iid* Gaussian random variables with mean zero and variance $N_0/2$. Terms α_j (for $j = 1, \ldots, m$) are m *iid* Rayleigh random variables, they are constant for all the bits forming each copy of the same information packet and independent on a packet copy basis. Hence, the decision variable $\tilde{Z}_i$ follows a Gaussian distribution. Moreover, the bit error probability of the BPSK modulation is:

$$P_e(m) = Q\left(\sqrt{2\, \psi(m)}\right) \quad (22)$$

where

$$\psi(m) \doteq \frac{E_b}{N_0} \sum_{j=1}^{m} \alpha_j^2 \quad (23)$$

Let us provide the definition of the probability density function of $\gamma(m)$. From [22] it can be easily proved that the $\gamma(m)$ is a χ^2-distributed random variable with $2m$ degrees of freedom. Hence, we have:

$$p\big(\psi(m)\big) = \frac{1}{(m-1)!\, \bar{\gamma}^m} \gamma(m)^{m-1} e^{-\frac{\gamma(m)}{\bar{\gamma}}} \quad (24)$$

where

$$\bar{\gamma} \doteq \frac{E_b}{N_0} E(\alpha_i^2) \quad (25)$$

with $E(\alpha_i^2)$ denoting the mean value of α_i [22]. We remark that the propagation conditions are assumed constant during the transmission interval of each of the m copies if the same information packet. As a result, the packet error probability of an information packet results to be:

$$P_B(m) = 1 - \int_0^{\infty} \Big[1 - Q\Big(\sqrt{2\, \gamma(m)}\Big)\Big]^L \cdot \cdot\, p\big(\gamma(m)\big)\, d\gamma(m) . \quad (26)$$

REFERENCES

1. Sakakibara K, Kasahara M. A multicast hybrid ARQ scheme using MDS codes and GMD decoding. *Communications, IEEE Transactions on* dec 1995; **43**(12):2933–2940.
2. Papadopoulos G, Koltsidas G, Pavlidou FN. Two hybrid ARQ algorithms for reliable multicast communications in UMTS networks. *Communications Letters, IEEE* apr 2006; **10**(4):260–262.
3. Farkas P, Yassine A, Tran V. Hybrid ARQ error control for broadcast or multicast in wireless networks. *Proc. of IWSSIP 2008*, 2008; 373–376.
4. Liu Z, Wu Z, Liu H, Wu M, Stein A. A Layered Hybrid-ARQ Scheme for Scalable Video Multicast Over Wireless Networks. *Proc. of ACSSC 2007*, 2007; 914–919.
5. Lacher M, Nonnenmacher J, Biersack E. Performance comparison of centralized versus distributed error recovery for reliable multicast. *Networking, IEEE/ACM Transactions on* apr 2000; **8**(2):224–238.

DOI: 10.1002/wcm
Prepared using wcmauth.cls

6. Wang J, Park SY, Love D, Zoltowski M. Throughput Delay Tradeoff for Wireless Multicast Using Hybrid-ARQ Protocols. *Communications, IEEE Transactions on* sep 2010; **58**(9):2741–2751.
7. Gopala P, Nam YH, El Gamal H. On the Error Exponents of ARQ Channels With Deadlines. *Information Theory, IEEE Transactions on* nov 2007; **53**(11):4265–4273.
8. El Heni N, Lagrange X, Maule P. Optimization of link adaptation and HARQ schemes for multicast in high speed cellular networks. *Proc. of ISWCS 2009*, 2009; 131–135.
9. Furht B, Ahson S. *HSDPA/HSUPA Handbook*. Internet and Communications, Taylor & Francis, 2010.
10. Lin TY, Lee SK, Tang HH, Lin MC. An Adaptive Hybrid ARQ Scheme with Constant Packet Lengths. *Communications, IEEE Transactions on* Oct 2012; **60**(10):2829–2840, doi:10.1109/TCOMM.2012.072412.100049A.
11. Fantacci R. Performance evaluation of efficient continuous ARQ protocols. *IEEE Trans. Commun.* Jun 1990; **38**(6):773–781, doi:10.1109/26.57469.
12. Kim J, Jin H, Sung DK, Schober R. Optimization of Wireless Multicast Systems Employing Hybrid-ARQ with Chase Combining. *Vehicular Technology, IEEE Transactions on* sep 2010; **59**(7):3342–3355.
13. Alexiou A, Asimakis K, Bouras C, Kokkinos V, Papazois A, Tseliou G. Reliable multicasting over LTE: A performance study. *Proc. of IEEE ISCC 2011*, 2011; 603–608, doi:10.1109/ISCC.2011.5983904.
14. Takahashi F, Higuchi K. HARQ for Predetermined-Rate Multicast Channel. *Vehicular Technology Conference (VTC 2010-Spring), 2010 IEEE 71st*, 2010; 1–5, doi:10.1109/VETECS.2010.5493786.
15. Wang J, Park SY, Love D, Zoltowski M. Throughput Delay Tradeoff for Wireless Multicast Using Hybrid-ARQ Protocols. *Communications, IEEE Transactions on* Sep 2010; **58**(9):2741–2751, doi:10.1109/TCOMM.2010.080310.080593.
16. Chase D. A Combined Coding and Modulation Approach for Communication over Dispersive Channels. *Communications, IEEE Transactions on* mar 1973; **21**(3):159–174.
17. Kemény J, Snell J. *Finite Markov Chains*. University series in undergraduate mathematics, Van Nostrand, 1960.
18. Chiti F, Fantacci R. A Soft Combining Hybrid-ARQ Technique Applied to Throughput Maximization Within 3G Satellite IP Networks. *Vehicular Technology, IEEE Transactions on* mar 2007; **56**(2):594–604.
19. Le Digabel S. Algorithm 909: NOMAD: Nonlinear Optimization with the MADS Algorithm. *ACM Trans. Math. Softw.* Feb 2011; **37**(4), doi:10.1145/1916461.1916468.
20. Abramson M, Audet C, Couture G, Dennis, Jr J, Le Digabel S, Tribes C. The NOMAD project. URL http://www.gerad.ca/nomad.
21. Fiedler M, Hlavacs H, Hackbarth K, Arlos P. Quality of experience. *Annals of Telecommunications* 2010; **65**:1–2.
22. Proakis J. *Digital Communications*. 4 edn., McGraw-Hill, 2000.
23. Brennan D. Linear diversity combining techniques. *Proceedings of the IEEE* feb 2003; **91**(2):331–356.