Reliable Multicast in LTE-A: An Energy Efficient Cross-layer Application of Network Coding

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***Abstract*—This paper presents a novel energy-aware communication scheme based on random network coding that is suitable for multicast and broadcast data delivery over Long Term Evolution (LTE) and LTE-Advanced networks. The proposed energy-aware transmission scheme minimises the average energy consumption of the macro base station that is required to deliver a message to all users in a multicast group in an LTE-A network. The energy saving gains and improved performance of the proposed scheme are compared to classical error control strategies. The reported analytical results clearly show a performance improvement of almost two-fold compared to the considered alternative.**

1. Introduction

3GPP standards have proposed several designs for the Multimedia Broadcast and Multicast Services (MBMS) framework in order to provide an efficient transmission strategy for Long Term Evolution (LTE) and LTE-Advanced (LTE-A) cellular networks. In particular, in Release 8 [1], the enhanced-MBMS (eMBMS) service is introduced with two proposed transmission schemes. The single-cell eMBMS (SC-eMBMS) transmission benefits from user feedback on channel conditions and the dynamic selection of suitable modulation and coding schemes. The multi-cell eMBMS or so called single frequency network e-MBMS (SFN-eMBMS) transmission scheme allows a group of neighboring cells to coordinate their transmissions to a user using the same physical signal [2].

The energy consumption of 3G and 4G cellular networks is mainly attributed to macro base stations (eNBs for short) and represents a major challenge to mobile network operators [3]. Nowadays, political and national initiatives currently support trends towards energy-saving in information technology and telecommunications, with the specific aim to lower the CO_2 emissions. Traditional solutions to reduce the energy demand are focused on reducing the eNB transmission power or even switching-off macro eNBs with low or no traffic load services [4]. These solutions should take into consideration the user Quality of Service (QoS) and the channel propagation conditions in order to avoid possible degradation in the user QoS with reduced energy usage. For example, in [5] the authors propose an energy-efficient optimization strategy that jointly considers the radio resource allocation process and the user admission control.

Reduced complexity Application Layer Random Linear Network Coding (AL-RLNC) solutions are currently proposed as alternative to the classical AL Forward Error Correction (AL-FEC) schemes [6] that are used for multimedia delivery in MBMS [7]. AL-RLNC solutions already provide reliable multimedia delivery over wireless networks [8]. However, given the large end-to-end delays between the application-layer entities compared to the short message transmission time, authors in [9] proposed the integration of the RLNC solution into the Medium Access Control (MAC) layer of the LTE/LTE-A RAN protocol stack. MAC-RLNC exploits the very small round-trip delay at the MAC layer to reduce the amount of redundancy produced by AL-RLNC. This makes MAC-RLNC a suitable solution for multimedia delivery over LTE-A [10].

In this paper, we extend the work presented in [9] to realise a fully reliable and energy-aware communication protocol that is suitable for eMBMS delivery over LTE-A. The proposed Extended-RLNC (E-RLNC) minimises the average energy consumption required to deliver a message to all users in a multicast group, by leveraging all available information on users' channel conditions and requested QoS to optimise the overall number of transmitted packets that ensure correct delivery of the transmitted message. To the best of our knowledge, this work offers the first detailed insight into the concept of energy reduction for eMBMS over LTE-A networks.

The paper is organized as follows. In Sec. 2 we provide a detailed description of E-RLNC integration within the LTE-A protocol stack, examining in detail all the packet-level operations starting from the Radio Link Control (RLC) layer down to the Physical Layer (PHY) layer. Sec. 3 describes the proposed energy-efficient optimization model for the E-RLNC scheme. The analysis results are presented in Sec. 4. Finally, we draw our conclusions in Sec. 5.

2. System Model

The integration of the RLNC communication scheme within the Medium Access Control (MAC) layer requires some changes to the LTE-A protocol stack. Fig. 1 shows the integration of the E-RLNC scheme in the LTE-A protocol structure and the cross-layer interactions to process a downlink data flow at the macro eNB side.

Let us consider a similar information flow to that in Fig. 1 that is directed to a Multicast Group (MG) composed of several mobile users (UEs). Starting at the Packet Data Conversion Protocol (PDCP) layer the Packet Data Units (PDUs) are

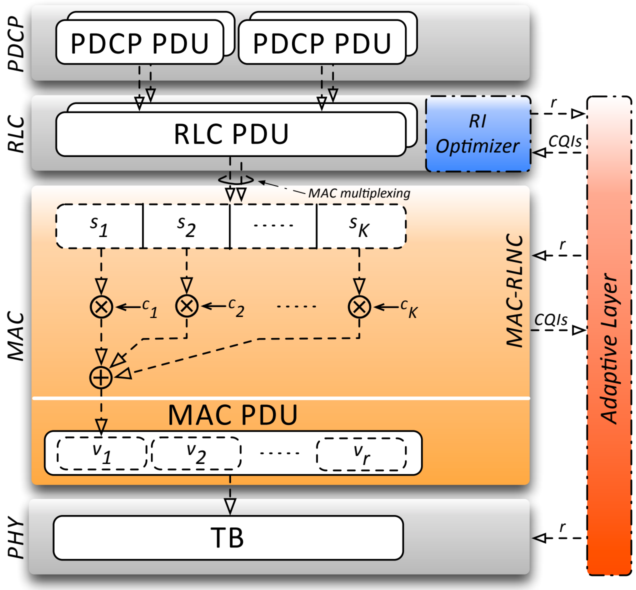

Fig. 1. E-RLNC optimized architecture for eNB and UE.

segmented/concatenated at the radio link control layer (RLC) to produce RLC PDUs of suitable length. A given RLC PDU directed to a MG will be kept in an appropriate buffer until all the UEs belonging to that MG will have successfully acknowledged (by an ACK message) the reception of the PDU itself. The RLC PDUs are then forwarded to the MAC layer. Each MAC Service Data Unit (SDU) represents the source message of the MAC-RLNC sublayer. This sublayer, proposed in [9], splits the MAC SDU into K equal-length source symbols (namely, $s_1, s_2, \dots, s_K$) and then encodes them in a RLNC fashion [11]. Basically, the coding process consists of randomly combining source symbols by using coefficients randomly selected within a finite field $\mathcal{F}_q$ (with size q). Let $\{s_1, s_2, \dots, s_K\}$ be the original information message of K symbols (i.e., an information message with generation length equal to K symbols) each of L bits. The output of the coding process (i.e., the output of the MAC-RLNC sublayer) is a stream of coded symbols $v_i = \sum_{j=1}^{K} c_j \cdot s_j$ (where c_j, for $j = 1, 2, \dots, K$, is the j-th coding vector).

Unlike [9], the protocol stack we are proposing in this paper is characterized by a MAC layer that generates a stream of MAC PDUs, where each PDU comprises r copies of the same coded symbol. All the MAC PDUs are forwarded to the PHY layer and mapped on different Transport Blocks[1] (TBs). Each UE can recover the original coded symbol (v_i) by soft-combining (according to the maximum likelihood principle) the r copies of v_i forming the MAC PDU. Whenever all the UEs have recovered the data message (i.e., when all the UEs has collected at least K linearly independent coded symbols), the eNB starts the transmission of the next one (it starts the transmission of the next MAC SDU).

[1]We assume here that a TB can hold just one MAC PDU.

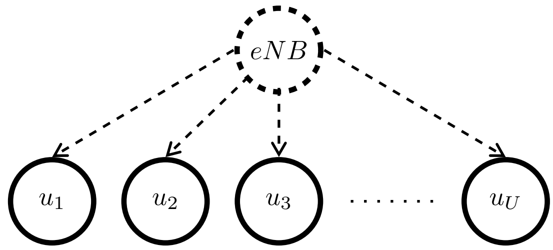

Fig. 2. Multicast network model.

In Sec. 3 will be discussed how to efficiently optimise the number of copies of the same coded symbols held by a TB. That is a key aspect in the energy efficient error control protocol we are proposing. If the r value is too small, the error probability of each coded symbol cannot be effectively reduced. However, as the value of r increases, the energy need to transmit a TB becomes bigger. The optimum value of r is a trade-off between the minimisation of the TB error probability and energy needed to broadcast the TB itself.

It is important to note that due to the fact that a multicast/broadcast communication flow adopts the Unacknowledged Mode at the RLC layer, it cannot adopt any classical error control strategy such as Automatic Repeat-reQuest (ARQ) or Hybrid ARQ (HARQ) strategies. However, [12] shows that an eNB can collect ACK messages from UEs targeted by MBMS services. This can be achieved for the UEs transmitting through the PUCCH channel that is commonly used in the case of the HARQ [13].

A key aspect of the error control strategy that we are proposing is represented by the optimization of the index r as a system parameter (i.e., a parameter shared among the eNB and the UEs belonging to the same MG). As will be described in the next section, the optimization algorithm should determine the best choice of r by considering the following inputs:

- the channel conditions reported by each UE, using the Channel Quality Indicator (CQI), in the MG
- the length L (expressed in bits) of each information symbol
- the Modulation and Coding (MC) scheme used to transmit a given MBMS flow.

In our model the optimization of the index r is in charge of the RLC layer as this layer is able to control and thus optimize different multicast communication flows independently (i.e., on a MG basis). Moreover, during the optimisation process the following cross-layer interactions are performed between RLC, MAC and PHY layers:

- the MAC and PHY layers have to know the index r in oder to provide a feasible allocation of TBs within subframes and, to correctly perform soft-combining of different copies of the same coded symbol, respectively;
- the MAC and RLC layers share all information related to the propagation conditions of the UEs.

The optimized r value and the reported CQI values of all UEs in the same MG are shared among layers of the communication stack by the adaptive layer [14].

3. Energy Efficient Optimization Model for the E-RLNC Scheme

A multicast communication in a MBMS network can be modeled efficiently as a classical Access Point (AP) where a node (namely, the eNB) transmits the same data to a set of devices (the UEs). Fig. 2 shows the network topology of a MG composed by U UEs. Let u_i (for $i = 1, \ldots, U$) be the i-th UE of a MG. Whenever u_i is able to recover the overall information message (transmitted according to the RLNC principle), it sends an acknowledgment message (ACK) to the eNB. The eNB continues to transmit coded symbols belonging to the same message until each UE of the MG has successfully recovered the information message (i.e., until the eNB has collected a number of ACKs equal to U).

The theoretical derivation proposed in the rest of the paper assumes that all the downlink transmissions directed to the members of a given MG adopt Quadrature Phase-Shift Keying (QPSK) modulation scheme[2].

The E-RLNC scheme described in section 2 is characterized by only one optimization variable: the index r. In this Section we define an energy efficient model for the optimization of r that is able to minimize the energy related to the transmission of a message. Moreover, in section 4 we show that the performance of the proposed heuristic optimisation model is close to that of the optimal optimisation model.

Let us consider again a given MG whose network topology is reported in Fig. 2. In addition to this, we have assumed that: (i) the losses of different TBs are statistically independent events, (ii) the propagation conditions are constant within a TB but are statistically independent between different downlink communication links. The i-th UE belonging to the considered MG receives TBs with the Signal-to-Noise Ratio (SNR) $\gamma_i = \alpha_i^2 \frac{E_b}{N_{0,i}}$ where: (i) α_i^2 is a random variable equal to the square of the Rayleigh channel coefficient, (ii) E_b is the energy associated to each symbol and, (iii) $N_{0,i}$ is the total noise power associated to each symbol received by the i-th UE.

Hence, the TB error probability characterizing the i-th UE can be given by:

$$P_i(r) = 1 - \int_0^\infty \Big[1 - p_i(r)\Big]^L f(\gamma_i) \ \ d\gamma_i = = 1 - \int_0^\infty \frac{1}{\overline{\gamma}_i} \Big[1 - p_i(r)\Big]^L e^{-\frac{1}{\overline{\gamma}_i}\gamma_i} \ \ d\gamma_i \quad (1)$$

where L is the coded symbol length (expressed in bits), $\overline{\gamma}_i$ is the average SNR (at the u_i side) and, the $f(\cdot)$ is the probability density function of γ_i. Finally, the term $p_i(r)$ defines the bit error probability (as function of the index r) affecting the reception at the u_i side. For the QPSK modulation the bit error probability is defined as [15]:

$$p_i(r) = \frac{1}{2}\,\mathrm{erfc}\Big(\sqrt{\frac{r\,\gamma_i}{2}}\Big) \ . \quad (2)$$

[2]It is important to note that the proposed analysis is general and can be easily extended to different modulation schemes, like 16-QAM or 64-QAM.

Let us assume that U_i (for $i = 1, \ldots, U$) is the number of TBs transmitted by the eNB until u_i has successfully recovered the information message, and that the random variable T gives the total number of TBs required to deliver the same message to all UEs in the MG. Hence, $T = \max_{i=1,\ldots,U} \Big\{U_i\Big\}$.

The i-th UE will be able to recover the information message within N transmissions if: (i) u_i will be able to successfully receive a given amount of error-free TBs (i.e., error-free coded symbols), (ii) these ones are linearly independent and, (iii) the ACK will be successfully received by the eNB. Hence, the probability that U_i is equal to or less than N (for $N \geq K$) is [16]:

$$F_i(N) = h_i(N) \sum_{a=K}^{N} \binom{N}{a} P_i^{N-a}(r) \Big[1 - P_i(r)\Big]^a g(a) \ . \quad (3)$$

The probability that an ACK message (transmitted by u_i) is successfully received by the eNB within N trials is

$$h_i(N) = 1 - P_{ack,i}^{N-K+1} \quad (4)$$

where $P_{ack,i}$ is the error probability of an ACK message transmitted by the i-th UE. The term $g(a)$ expresses the probability that at least K symbols over a, for $a \geq K$, are linearly independent and can be defined as [16]

$$g(a) = \prod_{t=0}^{K-1} \Big[1 - \frac{1}{q^{a-t}}\Big] \ . \quad (5)$$

We remark that q is the finite field size.

From the Eq. (3), the mean value of T is:

$$\overline{t}_{nc}(r) = \sum_{n=K}^{\infty} n \Bigg\{ \prod_{v=1}^{U} F_v(n) - \prod_{v=1}^{U} F_v(n-1) \Bigg\} \ . \quad (6)$$

Let $\hat{E}_b$ be the energy associated to the transmission of a TB using $r = 1$. The mean energy consumption $e_{nc}(r)$, normalized by $\hat{E}_b$ is

$$\overline{e}_{nc}(r) = r \ t_{nc}(r) \ . \quad (7)$$

For $r = 1$, Eq. (7) expresses the (normalized) mean energy consumption of an information message by using RLNC in the place of the E-RLNC.

The E-RLNC scheme improves energy efficiency by choosing the index r minimizing the mean energy consumption of a MG. This is the aim of the following optimization problem:

$$\text{(P1)} \qquad \text{minimize} \quad \overline{e}_{nc}(r) \quad (8)$$

$$\text{subject to} \quad r \in \mathbb{N} \setminus \{0\} \quad (9)$$

Due to the fact that P1 is a nonlinear problem, it is not feasible, from a computational point of view, to solve it in real time. In the rest of this Section we will propose an effective heuristic approach to overcome this problem.

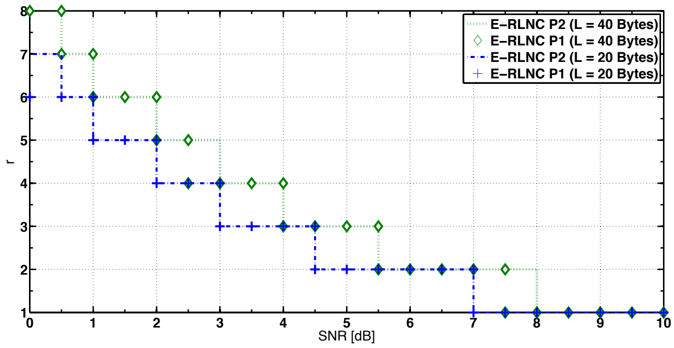

Fig. 3. The index r as function of $\overline{\gamma}_h$ value (scenario A).

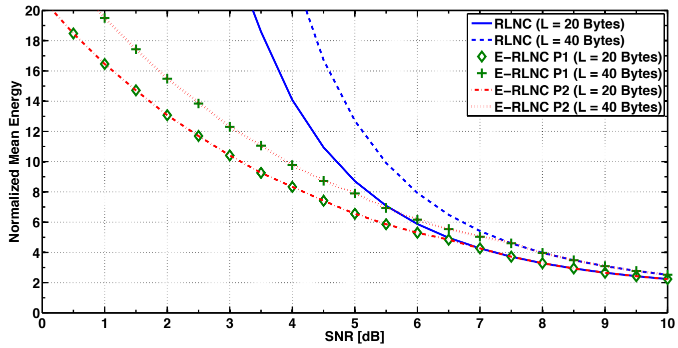

Fig. 4. The mean energy consumption as function of $\overline{\gamma}_h$ value (scenario A).

A. A Convex Heuristic Model

Let $\overline{Z}$ be the mean number of coded symbols (i.e., $\{v_1, v_2, \ldots, v_{\overline{Z}}\}$) that the eNB has to generate to successfully deliver a linearly independent set of *MAC PDUs*.

Let u_h be the UE of the MG experiencing the worst propagation conditions, we can approximate the mean energy consumption (normalized by $\hat{E}_b\,\overline{Z}$) as follows:

$$\bar{l}(r) = \frac{r}{1 - P_h(r)}\ . \tag{10}$$

As we can note, Eq. (10) approximates the Eq. (7).

Let $\tilde{l}(\tilde{r})$ be a function defined similar to Eq. (10) over a set of real numbers that are equal to or greater than one. Regardless of the value of $\overline{Z}$, from Eq. (10), we can define the following heuristic strategy for optimizing the index r:

$$\text{(P2)} \qquad \text{minimize} \quad \bar{l}(r), \tag{11}$$

$$\text{subject to} \quad r \in \mathbb{N} \setminus \{0\} \tag{12}$$

In order to solve P2, we can relax the constraint (12) and restate P2 in terms of a simple minimization of $\tilde{l}(\tilde{r})$. It is proved in the Appendix that $\tilde{l}(\tilde{r})$ is convex, hence, it can be minimized for that value of $\tilde{r}$ (namely, $\tilde{r}_o$) that is a solution of the following equation:

$$\frac{d}{d\,\tilde{r}}\left(\tilde{l}(\tilde{r})\right) = 0 \Leftrightarrow \tag{13}$$

$$\int_0^\infty \left[1 - \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{r\,\gamma_h}{2}}\right)\right]^L e^{-\frac{1}{\overline{\gamma}_h}\gamma_h}\, d\gamma_h +$$

$$-\frac{L\sqrt{\tilde{r}}}{2\sqrt{2\pi}}\int_0^\infty \sqrt{\gamma_h}\left[1 - \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{r\,\gamma_h}{2}}\right)\right]^{L-1} e^{-\frac{2+\tilde{r}\overline{\gamma}_h}{2\overline{\gamma}_h}\gamma_h}\, d\gamma_h = 0$$

Finally, the objective function (11) of P2 problem is minimized by that value of $r = r_o$ such that:

- $r_o = 1$, if (13) has no real root;
- $r_o = \mathrm{argmin}\{\bar{l}(r)\,|\,r = \lceil\tilde{r}_o\rceil, \lfloor\tilde{r}_o\rfloor\}$, otherwise.

4. Numerical Results

This section shows the numerical results of the proposed E-RLNC scheme by comparing the performance of the classical RLNC (i.e., the E-RLNC scheme with the index r equal to one) to the one of the E-RLNC optimized by resorting to the P1 or P2 optimization model. We remark that the P2 model represents a heuristic strategy able to provide a feasible but sub-optimal solution to the P1 problem.

The performance of the proposed E-RLNC scheme is investigated for a MG composed of a variable number of UEs. All the MBMS communication flows that adopt the RLNC or E-RLNC are characterised by a finite field of size $q = 2^8$, and a generation length $K = 20$. The probability of correct delivery of the ACK message to 99%. Finally, two information symbols lengths L equal to 20 and 40 bytes are used.

In order to inspect the performance of the E-RLNC over the classic RLNC and to show effectiveness of the proposed heuristic approach (i.e., the optimization model P2) over the optimization model P1, we considered the following scenarios:

- A: $U = 30$ UEs experience different channel conditions. The 1-st UE and the 30-th are characterized by $\overline{\gamma}_1 = s_o + \Delta$ dB and $\overline{\gamma}_{30} = 10 + \Delta$ dB, respectively. Moreover, $\overline{\gamma}_i = \overline{\gamma}_{i-1} + \Delta$ dB (for $i = 2, \ldots, U-1$). Finally, s_o takes values in the interval $[0, 10]$ dB and, $\Delta = \frac{10}{U-1}$ dB.
- B: U takes values in the $[2, 128]$ interval, $\overline{\gamma}_i = \tilde{\gamma}$ (for $i = 1, \ldots, U$). The parameter $\tilde{\gamma} = 3.5$ dB.

Let us focus on the network scenario A. Fig. 3 shows the optimum value of r as function of the value of $\overline{\gamma}_1$. The figure clearly shows that the indices of r selected by the proposed convex heuristic (i.e., P2) strategy typically overlaps those derived by the optimum model (i.e., P1), regardless of the chosen information symbol length. In particular, even if there are some slight differences, it does not have any impact on the overall system performance. This is clearly highlighted by Fig. 4 showing the normalized mean energy (of a single information symbol) as a function of the $\overline{\gamma}_1$ value. In addition to that, we can note that the E-RLNC clearly outperforms the classical RLNC strategy.

Let us consider network scenario B, where all of the UEs experience virtually the same propagation conditions. Fig. 5 shows the normalized mean energy consumption (on an information symbol basis) as a function of the $\tilde{\gamma}$. Here again, the performance of the proposed convex heuristic strategy is close to that of the optimum one. Moreover, also in this case we can note that the E-RLNC strategy outperforms the classical RLNC one.

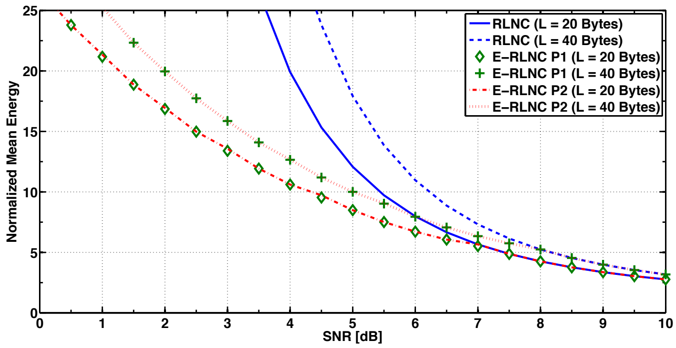

Fig. 5. The mean energy consumption as function of $\tilde{\gamma}$ value (scenario B, $U = 30$).

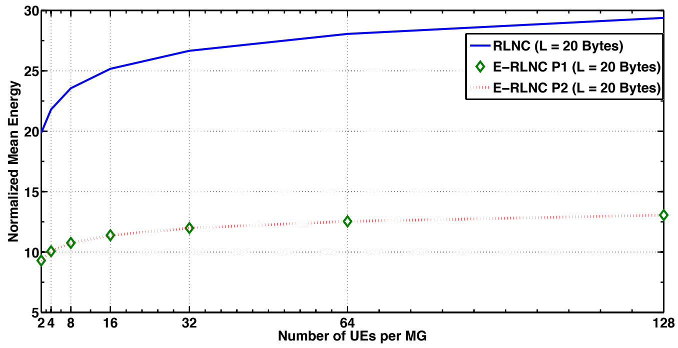

Fig. 6. The mean energy consumption as function of number of UEs belonging to the MG (scenario B).

Finally, let us consider Fig. 6 it shows the same performance metric as function of the U value. We can note that: (i) the performance gain of the heuristic strategy overlaps that of the optimum one, regardless to the U value and, (ii) the E-RLNC scheme is characterized by a performance gain of almost two-fold compared to the classical RLNC.

5. Conclusions

In this paper we propose an energy-efficient error control strategy for LTE/LTE-A networks based on MAC-RLNC scheme. We address in detail the problem of reducing the energy consumption required to deliver an information message to all users in a multicast group while maintaining a predefined quality of service for all those users. To solve this problem we propose two optimisation models an optimal model and a convex heuristic model. The numerical results show that the performance of the heuristic strategy is close to that of the optimal model, and more importantly that the proposed E-RLNC scheme significantly outperforms the classical RLNC scheme regardless under both optimisation models. E-RLNC Performance gain of almost two-fold is achieved compared to the classical RLNC scheme.

Appendix

Proposition 1: The function $\tilde{l}(\tilde{r})$ is convex on its domain.

Proof: The first-order derivative of $\tilde{l}(\tilde{r})$ can be expressed as follows:

$$\frac{d\,\tilde{l}(\tilde{r})}{d\,\tilde{r}} = \overline{\gamma}_h \, \frac{w(\tilde{r}) - t(\tilde{r})}{w^2(\tilde{r})} \tag{14}$$

where

$$w(\tilde{r}) \doteq \int_0^{\infty} \left[1 - \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{r\,\gamma_h}{2}}\right)\right]^L e^{-\frac{1}{\overline{\gamma}_h}\gamma_h}\, d\gamma_h$$

$$t(\tilde{r}) \doteq \frac{L\sqrt{\tilde{r}}}{2\sqrt{2\pi}} \int_0^{\infty} \sqrt{\gamma_h}\left[1 - \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{r\,\gamma_h}{2}}\right)\right]^{L-1} e^{-\frac{2+\tilde{r}\overline{\gamma}_h}{2\overline{\gamma}_h}\gamma_h}\, d\gamma_h$$

It can be easily verified that for $L \geq 8$ bits and $\overline{\gamma}_h \geq -1$ dB the relation $\frac{d^2\,\tilde{l}(\tilde{r})}{d\,\tilde{r}^2} \geq 0$ holds and $\frac{d\,\tilde{l}(\tilde{r})}{d\,\tilde{r}}$ increases. For these reasons $\tilde{l}(\tilde{r})$ is convex [17]. ■

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